Pre-Calculus

U-46 Curriculum Scope and Sequence

Reporting Strand	Instructional Focus	Common Core Standards	Sem.
Compose and Transform Functions	Compose and transform functions	F-BF-3(+), F-BF-1c	1
Inverses and Rational Functions	Produce inverse functions	F-BF- 4	1
	Graph and interpret rational functions	F-IF-7	
Exponential and Logarithmic Functions	Graph and interpret exponential and logarithmic functions	F-IF-7, F-BF-3(+)	1
	Use inverse relationships to solve problems	F-BF-5	
Series and Conics	Explore sequences	F.BF.1, F.IF.3	1/2
	Use finite and infinite formulas to solve problems	A-SSE-4 (+)	
	Derive the equations of ellipses and hyperbolas	G-GPE-2, G-GPE-3	
Unit Circle and Inverse Trigonometry	Use unit circles and inverse trigonometric functions	F-TF-3, F-TF-4, F-TF-6, F-TF-7	2
Graph and Transform Trigonometry	Graph and transform trigonometric functions	F-TF-4, F-BF-3(+), F-IF-7e	2
Prove and Use Trigonometry	Prove and use trigonometric functions	G-SRT-9, G-SRT.10, G-SRT.11, F-TF-9, F-TF-8	2
Limits and Coordinate Systems	Find limits and continuity	Calculus Prep	2
	Represent and calculate with vectors	N-VM-1, N-VM-2, N-VM-3, N-VM-4, N-VM-5	
	Represent and calculate complex numbers	N-CN-4, N-CN-3, N-CN-5, N-CN-6	

Functions

Instructional Focus: Compose and transform functions

	4 – Mastery	**3 – Proficient**	**2 - Basic**	**1 – Below Basic**	**0 – No Evidence**
Identify and Find Transfor-mations (F.BF.3)	Can extend thinking beyond the standard, including tasks that may involve one of the following: • Designing • Connecting • Synthesizing • Applying • Justifying • Critiquing • Analyzing • Creating • Proving	Identify the effect on a graph by replacing f(x) with more than two transformations: $f(x) + k$, $k\,f(x)$, $f(kx)$, $f(x + k)$ for specific positive and negative values of *k,* and graph the transformation Given the graph of a function and more than two transformations, find the values of the constants and coefficients Given a partial graph, complete the graph for both even and odd functions	Identify the effect on a graph by replacing f(x) with two transformations: $f(x) + k$, $k\,f(x)$, $f(kx)$, $f(x + k)$ for specific positive and negative values of *k,* and graph the transformation Given the graph of a function and two transformations, find the values of the constants and coefficients Recognize even and odd functions from graphs and equations	Identify the effect on a graph by replacing f(x) with a single transformation: $f(x) + k$, $k\,f(x)$, $f(kx)$, $f(x + k)$ for specific positive and negative values of k Given the graph of a function and a single transformation, find the value of the constant or coefficient Recognize even and odd functions from graphs or equations	Little evidence of reasoning or application to solve the problem Does not meet the criteria in a level 1
Compose Functions (F.BF.1c)		Evaluate the composition of 2 functions in context of a situation	Evaluate the composition of 2 functions	Evaluate a function for a given value and use that result to evaluate a second function	

F.BF.3 (+) Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of *k* (both positive and negative); find the value of *k* given the graphs. ~~Experiment with cases and illustrate an explanation of the effects on the graph using technology.~~ ***Include recognizing even and odd functions from their graphs and algebraic expressions for them.***

F.BF.1c Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.*

Functions

Instructional Focus: Produce inverse functions

	4 – Mastery	**3 – Proficient**	**2 - Basic**	**1 – Below Basic**	**0 – No Evidence**
Produce inverse functions (F.BF.4)	Can extend thinking beyond the standard, including tasks that may involve one of the following: • Designing • Connecting • Synthesizing • Applying • Justifying • Critiquing • Analyzing • Creating • Proving	Compose functions to verify if one function is the inverse of another function Read values of an inverse function from a graph and table **Produce an invertible function** from a non-invertible function by restricting the domain so that the function is one-to-one	**Compose functions** to verify if one function is the inverse of another function Read values of an inverse function from a graph **and** table **Identify a domain** that that will produce an invertible function from a non-invertible function	Given a simple function**,** **find its inverse** Read values of an inverse function from a graph **or** table **Identify if a function is invertible** from a graph	Little evidence of reasoning or application to solve the problem Does not meet the criteria in a level 1

F.BF.4 Find inverse functions.

b. (+) Verify by composition that one function is the inverse of another.

c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.

d. (+) Produce an invertible function from a non-invertible function by restricting the domain.

Functions

Instructional Focus: Graph and interpret rational functions

	4 – Mastery	3 – Proficient	2 - Basic	1 – Below Basic	0 – No Evidence
Identify key features of graphs (F.IF.7) The concentration *C* (in mg/dl), of a certain prescription drug in a person's bloodstream is determined using the rational function: $C(t)=\frac{50t}{t^2+25}$ where *t* is the time (in hours) after taking the prescription drug What is the equation of the horizontal asymptote for the graph of the function? What does this value (and the fact that it is an asymptote) represent in the context of this problem?	Can extend thinking beyond the standard, including tasks that may involve one of the following: • Designing • Connecting • Synthesizing • Applying • Justifying • Critiquing • Analyzing • Creating • Proving	Graph rational functions, given the model, and interpret all related key features of a graph **in context of a real world situation**. • equations of asymptotes • intercepts (x and y) • end behavior	**Graph** rational functions, given the model, and identify all related key features of a graph. • equations of asymptotes • intercepts (x and y) • end behavior	**Given the graphs** of rational functions, identify all related key features of a graph. • equations of asymptotes • intercepts (x and y) • end behavior	Little evidence of reasoning or application to solve the problem Does not meet the criteria in a level 1

F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★

d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

Exponential and Logarithmic Functions

Instructional Focus: Graph and interpret exponential and logarithmic functions

	4 – Mastery	**3 – Proficient**	**2 - Basic**	**1 – Below Basic**	**0 – No Evidence**
Identify and Find Transfor-mations (F.BF.3)	Can extend thinking beyond the standard, including tasks that may involve one of the following: • Designing • Connecting • Synthesizing • Applying • Justifying • Critiquing • Analyzing • Creating • Proving	Identify the effect on a graph by replacing f(x) with more than two transformations: *f*(*x*) + *k*, *k f*(*x*), *f*(*kx*), *f*(*x* + *k*) for specific positive and negative values of *k* Given the graph of a function and more than two transformations, find the values of the constants and coefficients	Identify the effect on a graph by replacing f(x) with two transformations: *f*(*x*) + *k*, *k f*(*x*), *f*(*kx*), *f*(*x* + *k*) for specific positive and negative values of *k* Given the graph of a function and two transformations, find the values of the constants and coefficients	Identify the effect on a graph by replacing f(x) with a single transformation: *f*(*x*) + *k*, *k f*(*x*), *f*(*kx*), *f*(*x* + *k*) for specific positive and negative values of k Given the graph of a function and a single transformation, find the value of the constant or coefficient	Little evidence of reasoning or application to solve the problem Does not meet the criteria in a level 1
Identify key features of graphs (F.IF.7)		Graph exponential and logarithmic functions, and interpret all related key features of a graph in context of a real world situation. • equations of asymptotes • intercepts (x and y) • end behavior	Graph exponential and logarithmic functions, and identify all related key features of a graph. • equations of asymptotes • intercepts (x and y) • end behavior	Given the graphs of exponential and logarithmic functions, and identify all related key features of a graph. • equations of asymptotes • intercepts (x and y) • end behavior	

F.BF.3 (+) Identify the effect on the graph of replacing *f*(*x*) by *f*(*x*) + *k*, *k f*(*x*), *f*(*kx*), and *f*(*x* + *k*) for specific values of *k* (both positive and negative); find the value of *k* given the graphs. ~~Experiment with cases and illustrate an explanation of the effects on the graph using technology.~~ *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*

F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★
d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
e. (+) Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

Exponential and Logarithmic Functions

Instructional Focus: Use inverse relationships to solve exponential and logarithmic problems

	4 – Mastery	**3 – Proficient**	**2 - Basic**	**1 – Below Basic**	**0 – No Evidence**
Exponential and Logarithmic inverses (F.BF.5)	Can extend thinking beyond the standard, including tasks that may involve one of the following: • Designing • Connecting • Synthesizing • Applying • Justifying • Critiquing • Analyzing • Creating • Proving	Recognize that exponential and logarithmic functions are inverses of each other and use these functions to solve **real-world problems**.	Recognize that exponential and logarithmic functions are inverses of each other **and use these functions to solve logarithmic and exponential equations**.	Recognize that exponential and logarithmic functions are inverses of each other and convert from one form into the other.	Little evidence of reasoning or application to solve the problem Does not meet the criteria in a level 1

F.BF.5 (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Series and Conics

Instructional Focus: Explore sequences

	4 – Mastery	**3 – Proficient**	**2 - Basic**	**1 – Below Basic**	**0 – No Evidence**
Recursive and Explicit Functions (F.BF.1a, F.IF.3, A.SSE.4)	Can extend thinking beyond the standard, including tasks that may involve one of the following: • Designing • Connecting • Synthesizing • Applying • Justifying • Critiquing • Analyzing • Creating • Proving	Write an explicit formula to model a situation in context. Use an explicit formula to find any term(s) in a sequence <u>given two non-consecutive terms.</u>	Write an explicit formula to model a situation <u>in context.</u> Use an explicit and recursive function <u>to find any term(s) in a sequence</u>.	Write an explicit and recursive function for an <u>arithmetic or geometric sequence.</u> <u>Identify characteristics</u> (first term, common ratio, etc) of an arithmetic or geometric sequence.	Little evidence of reasoning or application to solve the problem Does not meet the criteria in a level 1

Series and Conics

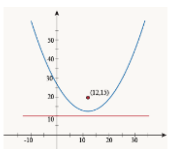
Instructional Focus: Use finite and infinite formulas to solve problems

	4 – Mastery	**3 – Proficient**	**2 - Basic**	**1 – Below Basic**	**0 – No Evidence**
Finite and infinite formulas (A.SSE.4)	Can extend thinking beyond the standard, including tasks that may involve one of the following: • Designing • Connecting • Synthesizing • Applying • Justifying • Critiquing • Analyzing • Creating • Proving	Use the finite and infinite formulas for geometric series to **solve real-world problems**	Use the finite and infinite formulas for geometric series to find: • sum • **first term** • **last term** • **rate**	**Find the sum**, using the finite and infinite formulas, for geometric series	Little evidence of reasoning or application to solve the problem Does not meet the criteria in a level 1

A.SSE.4 (edited) Use the finite **and infinite formulas** for geometric series to solve problems. For example, calculate mortgage payments. ★

Series and Conics

Instructional Focus: Derive the equation of ellipses and hyperbolas

	4 – Mastery	3 – Proficient	2 - Basic	1 – Below Basic	0 – No Evidence
Conics (G.GPE2, G.GPE.3) 	Can extend thinking beyond the standard, including tasks that may involve one of the following: • Designing • Connecting • Synthesizing • Applying • Justifying • Critiquing • Analyzing • Creating • Proving	**Write the equation** of a parabola given its focus and directrix. Write the standard equation of an ellipse or hyperbola given the graph, **foci, or general form of the equation**. Identify the center, vertices, **and foci** given the equation of an ellipse or hyperbola	**Identify the equation** of a parabola given its focus and directrix. **Write the standard equation of a hyperbola or ellipse given the graph** Identify the center **and vertices** of an ellipse or hyperbola given the graph or equation	Identify the focus and directix of a parabola **Identify** if a given equation represents an ellipse or hyperbola Identify the **center** of an ellipse or hyperbola given the graph or equation	Little evidence of reasoning or application to solve the problem Does not meet the criteria in a level 1

G.GPE.2 Derive the equation of a parabola given a focus and directrix.

G.GPE.3 (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.

Trigonometry

Instructional Focus: Use unit circles and inverse trigonometric functions

	4 – Mastery	3 – Proficient	2 - Basic	1 – Below Basic	0 – No Evidence
Use special triangles (F.TF.3)	Can extend thinking beyond the standard, including tasks that may involve one of the following: • Designing • Connecting • Synthesizing • Applying • Justifying • Critiquing • Analyzing • Creating • Proving	Use special triangles to determine the values of sine, cosine, tangent, secant, cosecant, and cotangent for 0, π/6, π/4 and π/3, π/2 and **use the unit circle to express the values of sine, cosine, tangent, secant, cosecant, and cotangent for π–*x*, π+*x*, and 2π–*x* in terms of their values for *x*, where *x* is any real number**	Use special triangles to determine the values of sine, cosine, tangent, **secant, cosecant, and cotangent** for **0**, π/6, π/4, π/3 and **π/2**	Use special triangles to determine the values of **sine, cosine and tangent** for π/6, π/4 and π/3	Little evidence of reasoning or application to solve the problem Does not meet the criteria in a level 1
Use unit circles to find values (F.TF.4)		Use the unit circle to express any angle, including negative angles and angles involving more than 1 rotation, in terms of its standard position to find **all six** trigonometric functions.	Use the unit circle to express any angle, between 0 and 2π, in terms of its standard position to find ALL 6 trig functions.	Use the unit circle to express any angle, between 0 and 2π, in terms of its standard position to find the **sine, cosine, and tangent functions**.	
Construct Inverse trigono-metric functions (F.TF.6)		**Construct an invertible trigonometric function by restricting the domain so that the function is always increasing or decreasing**	**Identify a domain that will allow construction of the inverse of a trigonometric function, because the function would be always increasing or decreasing**	Given a portion of a trigonometric graph, identify if that part of the graph is invertible	
Use inverse trigono-metric functions (F.TF.7)		Use inverse functions to solve trigonometric equations with restricted and unrestricted domains **and interpret the solutions in context of the situation**	Use inverse functions to solve trigonometric equations with restricted **and unrestricted** domains	Use inverse functions to solve trigonometric equations with **restricted domains**	
Pythagor-ean identity (F.TF.8) Given $\cos\theta =$		**Prove** the Pythagorean identity $sin^2(\theta) + cos^2(\theta) = 1$ and use it to find $sin(\theta)$, $cos(\theta)$, and $tan(\theta)$	Use the Pythagorean identity $sin^2(\theta) + cos^2(\theta) = 1$ to find $sin(\theta)$, $cos(\theta)$, **and** $tan(\theta)$	Use the Pythagorean identity $sin^2(\theta) + cos^2(\theta) = 1$ to find $sin(\theta)$, $cos(\theta)$, **or** $tan(\theta)$	

F.TF.3 (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosine, and tangent for π–*x*, π+*x*, and 2π–*x* in terms of their values for *x*, where *x* is any real number.
Functions F.TF.4 (+) Use the unit circle to ~~explain symmetry (odd and even)~~ and periodicity of trigonometric functions.

F.TF.6 (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.

F.TF.7 (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.★

F.TF.8 Prove the Pythagorean identity $sin^2(\theta) + cos^2(\theta) = 1$ and use it to find $sin(\theta)$, $cos(\theta)$,or $tan(\theta)$ given $sin(\theta)$, $cos(\theta)$, or $tan(\theta)$ and the quadrant of the angle.

Trigonometry

Instructional Focus: Graph and transform trigonometric functions

	4 – Mastery	**3 – Proficient**	**2 - Basic**	**1 – Below Basic**	**0 – No Evidence**
Symmetry and periodicity of trigono-metric functions (F.TF.4)	Can extend thinking beyond the standard, including tasks that may involve one of the following: • Designing • Connecting • Synthesizing • Applying • Justifying • Critiquing • Analyzing • Creating • Proving	Use the unit circle to explain symmetry (odd and even) of **<u>the six</u>** trigonometric functions. Use the periodicity of the unit circle to explain the repeated cycle of the graphs of **<u>all six</u>** trigonometric functions.	Use the unit circle to explain symmetry (odd and even) of the sine, cosine, **<u>and tangent</u>** functions. Use the periodicity of the unit circle to explain the repeated cycle of the graphs of sine, cosine, **<u>and tangent</u>** functions.	Use the unit circle to explain symmetry (odd and even) of the **<u>sine and cosine</u>** functions. Use the periodicity of the unit circle to explain the repeated cycle of the graphs of **<u>sine and cosine</u>** functions.	Little evidence of reasoning or application to solve the problem Does not meet the criteria in a level 1
Identify and Find Transfor-mations (F.BF.3)		Identify the effect on a graph by replacing f(x) with **<u>more than two</u>** transformations: $f(x) + k$, $k\,f(x)$, $f(kx)$, $f(x + k)$ for specific positive and negative values of k Given the graph of a function and **<u>more than two</u>** transformations, find the values of the constants and coefficients Given a partial graph, **<u>complete the graph</u>** for both even and odd functions	Identify the effect on a graph by replacing f(x) **<u>with two</u>** transformations: $f(x) + k$, $k\,f(x)$, $f(kx)$, $f(x + k)$ for specific positive and negative values of k Given the graph of a function and **<u>two transformations</u>**, find the values of the constants and coefficients Recognize even and odd functions from graphs **<u>and equations</u>**	Identify the effect on a graph by replacing f(x) with **<u>a single</u>** transformation: $f(x) + k$, $k\,f(x)$, $f(kx)$, $f(x + k)$ for specific positive and negative values of k Given the graph of a function and a **<u>single transformation</u>**, find the value of the constant or coefficient Recognize even and odd functions **<u>from graphs</u>**	
Identify key features of graphs (F.IF.7)		Graph trigonometric functions, and interpret all related key features of a graph **<u>in context of a real world situation</u>**. • asymptotes • period • midline • amplitude	**<u>Graph</u>** trigonometric functions, and identify all related key features of a graph. • asymptotes • period • midline • amplitude	**<u>Given the graph or equation</u>** of trigonometric functions, identify all related key features of a graph. • asymptotes • period • midline • amplitude	

Graphing F.TF.4 (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

F.BF.3 (+) Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. ~~Experiment with cases and illustrate an explanation of the effects on the graph using technology.~~ *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*

F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★

e. (+) Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

Trigonometry

Instructional Focus: Prove and use trigonometric functions

	4 – Mastery	3 – Proficient	2 - Basic	1 – Below Basic	0 – No Evidence
Prove and use formulas (F.TF.9)	Can extend thinking beyond the standard, including tasks that may involve one of the following: • Designing • Connecting • Synthesizing • Applying • Justifying • Critiquing • Analyzing • Creating • Proving	Prove the addition and subtraction formulas for sine, cosine, and tangent and use the addition and subtraction formulas to solve **identities**	**Prove the addition and subtraction formulas for sine, cosine, and tangent** and use them to solve numerical problems	Use the addition, subtraction, and tangent formulas to solve numerical problems	Little evidence of reasoning or application to solve the problem Does not meet the criteria in a level 1
Derive area formula (G.SRT.9)		Explain how to derive the formula: A = 1/2 ab sin(C) for the area of a triangle, and utilize it to find the area of a **polygon composed of multiple triangles**	**Explain how to derive the formula: A = 1/2 ab sin(C) for the area of a triangle**, and utilize it to find the area of a triangle	Find the area of any triangle using the formula: A = 1/2 ab sin(C)	
Law of Sines and Cosines (G.SRT.10 and 11)		Apply the Law of Sines and the Law of Cosines to find unknown measurements in oblique triangles **and interpret solutions in context of real-world situations**	Apply the Law of Sines **and** the Law of Cosines to find unknown measurements in oblique triangles	Apply the Law of Sines **or** the Law of Cosines to find unknown measurements in oblique triangles	

F.TF.9 (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

G.SRT.9 (+) Derive the formula A = 1/2 ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.

G.SRT.10 (+) Prove the Laws of Sines and Cosines and use them to solve problems.

G.SRT.11 (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Coordinate Systems

Instructional Focus: Represent and calculate with vectors

	4 – Mastery	3 – Proficient	2 - Basic	1 – Below Basic	0 – No Evidence
Represent vectors (N.VM.1)	Can extend thinking beyond the standard, including tasks that may involve one of the following: • Designing • Connecting • Synthesizing • Applying • Justifying • Critiquing • Analyzing • Creating • Proving	Use appropriate symbols for vectors and their magnitude, represent vector quantities by directed line segments, **and find the magnitude and direction of vector quantities.**	Use appropriate symbols for vectors and their magnitude and **represent vector quantities by directed line segments**.	Use appropriate **symbols for vectors and their magnitude**	Little evidence of reasoning or application to solve the problem Does not meet the criteria in a level 1
Solve problems with vectors (N.VM.3)		Solve problems involving velocity and other quantities by converting given direction and magnitude quantities into component vectors, calculate the resultant vector, **and find the resultant direction and magnitude or the angle between vectors**	Solve problems involving velocity and other quantities by converting given direction and magnitude quantities into component vectors**, and calculate the resultant vector**	Solve problems involving velocity and other quantities by **converting given direction and magnitude quantities into component vectors**	
Operations with vectors (N.VM.2, N.VM.4, N.VM.5)		Find the components of a vector by subtracting coordinates Add, subtract vectors graphically and component-wise, **and determine the magnitude and direction** Multiply a vector by a scalar and **determine the magnitude and direction**	Find the components of a vector by subtracting coordinates Add, subtract vectors graphically **and** component-wise Multiply a vector by a scalar	Find the components of a vector by subtracting coordinates Add, subtract vectors graphically **or** component-wise Multiply a vector by a scalar	

N.VM.1 (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., $\boldsymbol{v}$, $|\boldsymbol{v}|$, $||\boldsymbol{v}||$, v).

N.VM.3 (+) Solve problems involving velocity and other quantities that can be represented by vectors.

N.VM.2 (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.

N.VM.4 (+) Add and subtract vectors.

a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.

b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.

c. Understand vector subtraction $\boldsymbol{v} - \boldsymbol{w}$ as $\boldsymbol{v} + (-\boldsymbol{w})$, where $-\boldsymbol{w}$ is the additive inverse of $\boldsymbol{w}$, with the same magnitude as $\boldsymbol{w}$ and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.

N.VM.5 (+) Multiply a vector by a scalar.

a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$.

b. Compute the magnitude of a scalar multiple $c\boldsymbol{v}$ using $||c\boldsymbol{v}|| = |c|v$. Compute the direction of $c\boldsymbol{v}$ knowing that when $|c|v \neq 0$, the direction of $c\boldsymbol{v}$ is either along $\boldsymbol{v}$ (for $c > 0$) or against $\boldsymbol{v}$ (for $c < 0$).

Coordinate Systems

Instructional Focus: Represent and calculate complex numbers

	4 – Mastery	3 – Proficient	2 - Basic	1 – Below Basic	0 – No Evidence
Represent on the complex plane (N.CN.4)	Can extend thinking beyond the standard, including tasks that may involve one of the following: • Designing • Connecting • Synthesizing • Applying • Justifying • Critiquing • Analyzing • Creating • Proving	Represent complex numbers on the complex plane in rectangular and polar form, **and explain** why the rectangular and polar forms of a given complex number represent the same number	Represent complex numbers on the complex plane in rectangular **and polar form**	Represent complex numbers on the complex plane in **rectangular form**	Little evidence of reasoning or application to solve the problem Does not meet the criteria in a level 1
Operations of Vectors in Polar Form (N.CN.3, N.CN.5)		Represent and compute addition and subtraction of complex numbers geometrically on the complex plane Represent and compute multiplication and division, in polar form, of complex numbers geometrically on the complex plane **Represent and compute the power and roots of complex numbers, in polar form.**	Represent and compute addition and subtraction of complex numbers geometrically on the complex plane **Represent and compute multiplication and division, in polar form, of complex numbers geometrically on the complex plane**	Represent and **compute addition and subtraction** of complex numbers geometrically on the complex plane	
Calculate distance and midpoint (N.CN.6)		Calculate the **distance between numbers in the complex plane as the modulus of the difference**, and calculate the midpoint of a segment in the complex plane as the average of the numbers at its endpoints	Calculate the **difference between numbers in the complex plane**, and calculate the midpoint of a segment in the complex plane as the average of the numbers at its endpoints	Calculate the **midpoint of a segment in the complex plane** as the average of the numbers at its endpoints	

N.CN.4 (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.

N.CN.3 (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

N.CN.5 (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example,* $(-1 + \sqrt{3}\,i)^3 = 8$ *because* $(-1 + \sqrt{3}\,i)$ *has modulus* 2 *and argument* 120°.

N.CN.6 (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.

Limits

Instructional Focus: Find limits and continuity

<table>
<tr><th></th><th>4 – Mastery</th><th>3 – Proficient</th><th>2 - Basic</th><th>1 – Below Basic</th><th>0 – No Evidence</th></tr>
<tr><td>Find limits</td><td rowspan="2">Can extend thinking beyond the standard, including tasks that may involve one of the following:

• Designing
• Connecting
• Synthesizing
• Applying
• Justifying
• Critiquing
• Analyzing
• Creating
• Proving</td><td>Find limits and one-sided limits graphically, numerically**, and algebraically, using proper notation.** Describe end behavior (as x approaches or -) using limit notation</td><td>Find limits and one-sided limits graphically and numerically. **Describe end behavior (as x approaches ∞ or $-\infty$) using limit notation).**</td><td>Find **limits and one-sided limits** graphically and numerically</td><td rowspan="2">Little evidence of reasoning or application to solve the problem

Does not meet the criteria in a level 1</td></tr>
<tr><td>Determine continuity</td><td>Determine continuity of functions graphically, numerically, **and algebraically on its domain** using the three-part definition of continuous functions.

Determine values for which a function is discontinuous, understand the difference between removable and nonremovable discontinuities, **and be able to redefine functions to make them continuous when possible.**

Find finite and infinite one-sided limits, **and describe asymptotes using limit notation.**</td><td>Determine continuity of functions graphically and numerically **on its domain** using the three-part definition of continuous functions.

Determine values for which a function is discontinuous, **and understand the difference between removable and nonremovable discontinuities.**

Find finite and infinite one-sided limits.</td><td>Determine continuity of functions graphically and numerically **at a given value** using the three-part definition of continuous functions.

Determine **values for which a function is discontinuous.**

Determine whether a one-sided limit is finite or infinite.</td></tr>
</table>

Find limits and one-sided limits graphically, numerically, and algebraically, using proper notation. Describe end behavior (as x approaches or -) using limit notation.
Determine continuity of functions graphically, numerically, and algebraically on its domain using the three-part definition of continuous functions. Determine values for which a function is discontinuous, understand the difference between removable and nonremovable discontinuities, and be able to redefine functions to make them continuous when possible. Find finite and infinite one-sided limits, and describe asymptotes using limit notation.